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Kinetic Equation for Particle Transport and Heat Transfer in Nonisothermal Turbulent Flows

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The physical situation of two-phase nonisothermal turbulent fluid flows laden with nonevaporating spherical particles is considered. A closed kinetic equation for the transport of particles and their temperature is derived by solving the involved closure problem using the functional method. The equation is exact when the fluctuations of fluid flow variables along the particle path are distributed as Gaussian. For the case of homogeneous turbulent flows, macroscopic equations describing the time evolution of statistical properties related to particle velocity and temperature are obtained by taking various moments of the closed kinetic equation. These equations are computed for the case of homogeneous shear turbulent flow with constant temperature gradient, and the results are compared with the data of our direct numerical simulation study.

Nomenclature

C_l	=	specific heat of particle
C_p	=	specific heat of carrier fluid
d	=	diameter of particle
G_j	=	generalized response function [Eq. (26)]
G_{jk}	=	generalized response function [Eq. (25)]
G^θ	=	generalized response function [Eq. (27)]
h	=	phase space heat current
$\boldsymbol{j}$	=	phase space diffusion current
k	=	turbulence kinetic energy
L_f	=	reference length
N	=	average number density of particle phase
Pr	=	Prandtl number
Re_d	=	particle Reynolds number
Re_f	=	reference Reynolds number $(\rho_f U_f L_f/\mu)$
T	=	fluid temperature
T_d	=	particle temperature
T_f	=	reference temperature
$\bar{T}$	=	ensemble average of fluid temperature
$\tilde{T}_{b_i b_j}$	=	Lagrangian integral time scale
t	=	time
$t'(\boldsymbol{x}, t)$	=	temperature fluctuation of fluid
$\boldsymbol{U}$	=	velocity of fluid
U_f	=	reference velocity
$\bar{U}_1$	=	ensemble average of $\boldsymbol{U}$ component in the direction of x_1
$\boldsymbol{u}_0$	=	uniform translational velocity
$\boldsymbol{u}'$	=	velocity fluctuation of fluid
u'_j	=	jth component of $\boldsymbol{u}'$
$\boldsymbol{V}$	=	particle velocity
$\bar{V}_i$	=	ith component of density average velocity of particle phase
$\boldsymbol{v}$	=	phase space variable for particle velocity
W	=	phase space density
$\boldsymbol{X}$	=	particle position
$\boldsymbol{x}$	=	phase space variable for particle position
α	=	$\partial \bar{U}_1/\partial x_2$
$\beta_{b_i b_j}$	=	$1/\tilde{T}_{b_i b_j}$
β_v	=	f_1/τ_d
β_θ	=	f_2/τ_d
β'_v	=	fluctuating part of β_v
β'_θ	=	fluctuating part of β_θ
ϵ	=	rate of turbulence viscous dissipation
$\bar{\Theta}$	=	density average temperature of particle phase
θ	=	phase space variable for particle temperature
θ_0	=	uniform temperature
θ'	=	fluctuation in temperature of particle phase
μ	=	viscosity of fluid
ρ_d	=	particle mass density
ρ_f	=	reference density
ρ^*	=	density of fluid at particle location
σ	=	C_l/C_p
τ_d	=	particle time constant

Introduction

PARTICLE-/DROPLET-LADEN turbulent flows occur in many important natural and technological situations, for example, clouds,[1] aerosol transport and deposition,[2] spray combustion,[3,4] fluidized bed combustion,[5] plasma spray coating, and synthesis of nanoparticles.[6] Undoubtedly, turbulence itself remains as a difficult and unsolved problem of classical mechanics, despite many persistent efforts by physicists and engineers. The presence of particles/droplets (hereafter simply referred to as particles) further adds to the complexity of turbulence. The statistical descriptions of fluid phase and dispersed phase of particles are required to understand and predict such flows. In this paper, we focus on the statistical modeling of the dispersed phase of particles in nonisothermal turbulent flows.

Two main approaches are presently available for statistical modeling of a turbulent dispersed phase: 1) Reynolds-averaged Navier–Stokes (RANS)-type closure modeling[7,8] and 2) kinetic equation or probability density function (PDF) modeling.[9] In both of these approaches, particles are considered as point particles and the analysis begins from the Lagrangian equations governing the particle trajectory and its other properties, such as temperature. In a RANS-type approach, instantaneous equations for particle concentration, velocity, and other properties of interest are formed from the Lagrangian equations.[10,11] These equations are in the Eulerian framework, and their ensemble average represents the transport equations for mean values of the properties. The process of ensemble averaging generates unknown correlations that are modeled using the techniques available for RANS modeling of the fluid phase.

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In recent years, the second approach of kinetic equation, or PDF modeling, has evolved through the works of Reeks,[12,13] Hyland et al.,[14] Derevich and Zaichik,[15] Derevich,[16] Zaichik,[17] and Pozorski and Minier.[18] In the PDF approach, by the use of the Liouville theorem and Lagrangian equations for the particle, an equation is formed for the phase space density W. The ensemble average of this equation represents transport of the PDF in the phase space, which includes variables such as particle position, velocity, temperature, etc. It is possible to write a PDF equation with fluid flow variables included in the list of phase-space variables. Such a type of general approach is discussed in greater detail by Minier and Peirano.[9] Here, we focus on obtaining a PDF equation for particle properties by considering fluid properties as external and known variables. In such a case, the ensemble-averaged equation contains unknown correlations between the fluctuating part of W and the fluctuating variables of the fluid flow and, thus, pose closure problems.

Different methods of solving the closure problems appearing in PDF transport equations have been utilized by Reeks,[13,19] Hyland et al.,[14] Zaichik,[17] Derevich,[16] Pozorski and Minier,[18] and Pandya and Mashayek.[20] These methods are, namely, Kraichnan's direct interaction approximation (DIA),[21] the Lagrangian history direct interaction (LHDI) approximation[22] functional method, and Van Kampen's method.[23–25] The functional method gives an exact solution when the fluctuations involved in the problem are Gaussian in nature. The DIA, LHDI, and Van Kampen's method are applicable even when the fluctuations distributions are not Gaussian. Reeks[12,13,19] showed that LHDI is superior to DIA because LHDI preserves the symmetry of the phenomena under random Galilean transformation. The closed PDF equation obtained through LHDI was recently rederived by the use of Van Kampen's method by Pozorski and Minier[18] and by the use of the functional method by Hyland et al.[14] Also, the closed kinetic equations are useful in deriving macroscopic equations and constitutive relations for the statistical properties of the dispersed phase.[13,16,17] These macroscopic equations are in the Eulerian framework and are easier to compute than the PDF equation.

In this paper, we consider nonisothermal, spherical monodispersed particles that move under the action of fluid drag force and exchange heat with the surrounding turbulent fluid flow. We consider fluid variables, namely, velocity and temperature, as external variables. Then, the goal is to obtain a transport equation for the PDF describing the distribution of particle position, velocity, and temperature at time t. The closure problems are first discussed in the next section in the framework of the kinetic approach. Then, a closed kinetic or PDF equation is derived after solving the problems by using the functional methodology of Hyland et al.[14] Then an ideal situation of nonisothermal flow is considered. It is suggested that the exact solution in this ideal situation can be taken as a first test for assessing other closure schemes. Furthermore, macroscopic equations are derived for homogeneous shear flow with uniform temperature gradient. These equations are then solved, and the results are compared with the direct numerical simulation (DNS) data. Finally, some concluding remarks are provided.

Closure Problems

Consider a two-phase nonisothermal turbulent flow in which collisionless spherical particles with diameter d move under the influence of stochastic fluid drag force and their temperature T_d changes due to the thermal interaction [driven by the temperature difference $(T - T_d)$] with the carrier fluid. Here T is the temperature of the fluid in the vicinity of the particle. The Lagrangian equations governing the time variation of the position $\boldsymbol{X}$, velocity $\boldsymbol{V}$, and temperature T_d of the particle at time t are

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \boldsymbol{V} \tag{1}$$

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \beta_v(\boldsymbol{U} - \boldsymbol{V}) \tag{2}$$

$$\frac{\mathrm{d}T_d}{\mathrm{d}t} = \beta_\theta(T - T_d) \tag{3}$$

where

$$\beta_v = \frac{f_1}{\tau_d}, \qquad \beta_\theta = \frac{f_2}{\tau_d}, \qquad \tau_d = \frac{Re_f \rho_d d^2}{18} \tag{4}$$

$$f_1 = 1 + 0.15 Re_d^{0.687}, \qquad f_2 = \frac{2 + 0.6 Re_d^{0.5} Pr^{0.33}}{3 Pr \sigma} \tag{5}$$

$$Re_d = Re_f \rho^* d |\boldsymbol{U} - \boldsymbol{V}|, \qquad Re_f = \frac{\rho_f U_f L_f}{\mu} \tag{6}$$

The terms containing Re_d in the expressions for f_1 and f_2 account for the corrections to the Stokes drag and heat transfer when Re_d is of the order one and larger. All of the flow variables are nondimensionalized by L_f, ρ_f, U_f, and T_f scales.

By the use of the Lagrangian equations (1–3) and the Liouville theorem, an equation for the phase space density[14,26] $W(\boldsymbol{x}, \boldsymbol{v}, \theta, t) = \delta(\boldsymbol{X} - \boldsymbol{x})\delta(\boldsymbol{V} - \boldsymbol{v})\delta(T_d - \theta)$ can be written as

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}} \cdot \boldsymbol{v} + \frac{\partial}{\partial \boldsymbol{v}} \cdot \beta_v(\boldsymbol{U} - \boldsymbol{v}) + \frac{\partial}{\partial \theta}\beta_\theta(T - \theta)\right] W = 0 \tag{7}$$

where δ represents the Dirac delta function. The ensemble average (denoted by $\langle\,\rangle$) of Eq. (7) represents the transport equation for the PDF and is

$$\begin{aligned}
&\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}} \cdot \boldsymbol{v} + \frac{\partial}{\partial \boldsymbol{v}} \cdot \langle\beta_v\rangle(\bar{\boldsymbol{U}} - \boldsymbol{v}) + \frac{\partial}{\partial \theta}\langle\beta_\theta\rangle(\bar{T} - \theta)\right]\langle W\rangle \\
&\quad = -\frac{\partial}{\partial \boldsymbol{v}} \cdot \langle\beta_v\rangle\langle \boldsymbol{u}' W\rangle - \frac{\partial}{\partial \theta}\langle\beta_\theta\rangle\langle t' W\rangle \\
&\quad - \left[\frac{\partial}{\partial \boldsymbol{v}} \cdot (\bar{\boldsymbol{U}} - \boldsymbol{v})\langle\beta_v' W\rangle + \frac{\partial}{\partial \theta}(\bar{T} - \theta)\langle\beta_\theta' W\rangle\right. \\
&\quad \left. + \frac{\partial}{\partial \boldsymbol{v}} \cdot \langle\beta_v' \boldsymbol{u}' W\rangle + \frac{\partial}{\partial \theta}\langle\beta_\theta' t' W\rangle\right]
\end{aligned} \tag{8}$$

with

$$\beta_v = \langle\beta_v\rangle + \beta_v', \qquad \beta_\theta = \langle\beta_\theta\rangle + \beta_\theta' \tag{9}$$

In Eq. (8), $\bar{\boldsymbol{U}}$ and $\bar{T}$ are the average parts and $\boldsymbol{u}'$ and t' are the fluctuating parts of the velocity and temperature of the fluid in the vicinity of particle, respectively. Also, β_v' and β_θ' represent fluctuations in β_v and β_θ over their average value $\langle\beta_v\rangle$ and $\langle\beta_\theta\rangle$, respectively, and can be written as

$$\beta_v' = \frac{0.15\left[Re_d^{0.687} - \left\langle Re_d^{0.687}\right\rangle\right]}{\tau_d} \tag{10}$$

$$\beta_\theta' = \frac{0.6 Pr^{0.33}\left[Re_d^{0.5} - \left\langle Re_d^{0.5}\right\rangle\right]}{3\tau_d Pr\sigma} \tag{11}$$

where the terms in the square brackets of Eqs. (10) and (11) represent fluctuating parts of $Re_d^{0.687}$ and $Re_d^{0.5}$, respectively.

Various unknown correlations between W, $\boldsymbol{u}'$, t', β_v', and β_θ', which appear in different terms on the right-hand side (RHS) of Eq. (8), pose closure problems. In this paper, we provide closed expressions for correlations $\langle\boldsymbol{u}' W\rangle$ and $\langle t' W\rangle$ and neglect the correlations inside the square bracket on the RHS of Eq. (8), which include β_v' and β_θ'. This neglect is justified for particles with small values of τ_d because the correction terms themselves in the expressions for f_1 and f_2 are negligible. For the particles with larger values of τ_d, unknown terms with β_v' and β_θ' can contribute to the drift and diffusion of $\langle W\rangle$ in the phase space. Here we do not perform a complex exercise to obtain the drift velocity and diffusion due to β_v' and β_θ'. Rather, we keep the modeling simple. Its range of validity with regard to the values of

τ_d can be assessed through DNS data. Thus, we consider $\beta_v \cong \langle\beta_v\rangle$ and $\beta_\theta \cong \langle\beta_\theta\rangle$. In view of the same,

$$f_1 = 1 + 0.15\langle Re_d\rangle^{0.687} \tag{12}$$

$$f_2 = \frac{2 + 0.6\langle Re_d\rangle^{0.5}Pr^{0.33}}{3Pr\sigma} \tag{13}$$

$$\langle Re_d\rangle \cong Re_f\rho^* d[\langle|\boldsymbol{U} - \boldsymbol{V}|^2\rangle]^{0.5} \tag{14}$$

implying that the effect of fluctuations in Re_d are neglected too. Under these conditions, the first two unknown terms, present on the RHS of Eq. (8), containing $\boldsymbol{j} = \beta_v\langle\boldsymbol{u}'W\rangle$ and $h = \beta_\theta\langle t'W\rangle$, pose the closure problems. In principle, if we also include the fluid velocity $\boldsymbol{U}$ along the particle path in the phase space variables for W, then β_v and β_θ are not stochastic functions. The closure problems related to $\beta_v\langle\boldsymbol{u}'W\rangle$, and terms containing β_v' and β_θ', will not appear. However, in that case, we need another stochastic differential equation that describes the fluid velocity along the particle path.[18]

Closed Kinetic Equation

The closure problems appearing in the equation for $\langle W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)\rangle$ can be tackled by renormalized perturbation theories such as direct interaction approximation,[21] LHDI,[22] and Van Kampen's method.[25] Yet another method, based on the Furutsu–Novikov–Donsker formula, can be used to solve the closure, which obtains an exact solution when the fluctuations are Gaussian (see Ref. 14). Here, we assume $\boldsymbol{u}'$ and t' to be Gaussian and use the Furutsu–Novikov–Donsker formula for solving the involved closure problems.

In general situations of two-phase flows, the assumption of Gaussian distribution for fluctuations is not strictly valid. However, the results derived here for the closed expressions for $\boldsymbol{j}$ and h would serve as a first test for other closure schemes. That is, other closure schemes should reproduce the results derived here in the limiting case when the fluctuations are Gaussian in nature. The Furutsu–Novikov–Donsker formula suggests that, for any Gaussian random function $f_i(p)$, its correlation with functional $R[\boldsymbol{f}]$ can be exactly given by[14,27]

$$\langle f_i(p)R[\boldsymbol{f}]\rangle = \int \langle f_i(p)f_j(p')\rangle\left\langle\frac{\delta R[\boldsymbol{f}]}{\delta f_j(p')\,\mathrm{d}p'}\right\rangle \mathrm{d}p' \tag{15}$$

where the integral extends over the region of arguments p in which the function f_i is defined and $\langle\delta R[\boldsymbol{f}]/\delta f_j(p')\,\mathrm{d}p'\rangle$ represents the functional derivative of R with respect to f_j. This formula forms the basis of the Derevich,[16] Zaichik,[17] and Hyland et al.[14] work on the derivation of closed kinetic or PDF equations for a turbulent particle phase. Here, we use the framework of Hyland et al.[14] for isothermal flows and extend it to the present case of nonisothermal two-phase flows.

The functional dependence of W on $\boldsymbol{u}'$ and t' and the application of the Furutsu–Novikov–Donsker formula allow us to write

$$\langle u_i'W\rangle = \int\left[\langle u_i'(\boldsymbol{x}, t)u_j'(\boldsymbol{x}_1, t_1)\rangle\left\langle\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right\rangle + \langle u_i'(\boldsymbol{x}, t)t'(\boldsymbol{x}_1, t_1)\rangle\left\langle\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right\rangle\right]\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1 \tag{16}$$

$$\langle t'W\rangle = \int\left[\langle t'(\boldsymbol{x}, t)u_j'(\boldsymbol{x}_1, t_1)\rangle\left\langle\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right\rangle + \langle t'(\boldsymbol{x}, t)t'(\boldsymbol{x}_1, t_1)\rangle\left\langle\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right\rangle\right]\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1 \tag{17}$$

Here,

$$\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}, \qquad \frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}$$

are functional derivatives and are given by

$$\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = -\left[\frac{\partial}{\partial x_k}\frac{\delta X_k(t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} + \frac{\partial}{\partial v_k}\frac{\delta V_k(t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} + \frac{\partial}{\partial\theta}\frac{\delta T_d(t)}{\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right]W \tag{18}$$

$$\frac{\delta W(\boldsymbol{x}, \boldsymbol{v}, \theta, t)}{\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = -\left[\frac{\partial}{\partial\theta}\frac{\delta T_d(t)}{\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1}\right]W \tag{19}$$

While deriving Eqs. (18) and (19), we have used the following property for the derivative of any function $f(x - y)$ (Ref. 14):

$$\frac{\partial f(x - y)}{\partial x} = -\frac{\partial f(x - y)}{\partial y} \tag{20}$$

The functional derivatives of $X_k(t)$, $V_k(t)$, and $T_d(t)$ appearing in Eqs. (18) and (19) are written as

$$\frac{\delta X_k(t)}{\beta_v\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = G_{jk}[\boldsymbol{x}_1, t_1; \boldsymbol{X}(t), t]\,\delta\,[\boldsymbol{X}(t_1) - \boldsymbol{x}_1] \tag{21}$$

$$\frac{\delta V_k(t)}{\beta_v\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = \frac{\mathrm{d}}{\mathrm{d}t}G_{jk}[\boldsymbol{x}_1, t_1; \boldsymbol{X}(t), t]\,\delta\,[\boldsymbol{X}(t_1) - \boldsymbol{x}_1] \tag{22}$$

$$\frac{\delta T_d(t)}{\beta_v\delta u_j'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = G_j[\boldsymbol{x}_1, t_1; T_d(t), t]\,\delta\,[\boldsymbol{X}(t_1) - \boldsymbol{x}_1] \tag{23}$$

$$\frac{\delta T_d(t)}{\beta_\theta\delta t'(\boldsymbol{x}_1, t_1)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}t_1} = G^\theta[\boldsymbol{x}_1, t_1; T_d(t), t]\,\delta\,[\boldsymbol{X}(t_1) - \boldsymbol{x}_1] \tag{24}$$

where the generalized response functions G_{jk}, G_j, and G^θ are defined as

$$G_{jk}[\boldsymbol{X}_1, t_1; \boldsymbol{X}(t), t] = \frac{\delta X_k(t)}{\beta_v\delta u_j'(\boldsymbol{X}_1, t_1)\,\mathrm{d}t_1} \tag{25}$$

$$G_j[\boldsymbol{X}_1, t_1; T_d(t), t] = \frac{\delta T_d(t)}{\beta_v\delta u_j'(\boldsymbol{X}_1, t_1)\,\mathrm{d}t_1} \tag{26}$$

$$G^\theta[\boldsymbol{X}_1, t_1; T_d(t), t] = \frac{\delta T_d(t)}{\beta_\theta\delta t'(\boldsymbol{X}_1, t_1)\,\mathrm{d}t_1} \tag{27}$$

and their equations can be obtained from the Lagrangian equations (1–3), written as

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}G_{jk} + \beta_v\frac{\mathrm{d}}{\mathrm{d}t}G_{jk} - \beta_v G_{ji}\frac{\partial\bar{U}_k}{\partial X_i} = \delta_{jk}\delta(t - t_1) + A_{jk}\delta(t) \tag{28}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}G_j - \beta_\theta G_{jk}\frac{\partial\bar{T}}{\partial X_k} + \beta_\theta G_j = 0 \tag{29}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}G^\theta + \beta_\theta G^\theta = \delta(t - t_1) + C_\theta\delta(t) \tag{30}$$

These equations suggest that G_{jk}, G_j, and G^θ are statistically sharp functions. Here A_{jk} accounts for the correlation between particle velocity and fluid velocity at initial time $t = 0$, and C_θ accounts for the correlation between particle temperature and fluid temperature at the initial time. At time $t = 0$, if $\boldsymbol{V} = a_v\boldsymbol{U}$ and $T_d = a_\theta T$, then $A_{jk}(t_1)$ and $C_\theta(t_1)$ can be written as

$$A_{jk}(t_1) = (a_v/\beta_v)\delta_{jk}\delta(t_1), \qquad C_\theta = (a_\theta/\beta_\theta)\delta(t_1) \tag{31}$$

where a_v and a_θ are constants.

The expressions for $\boldsymbol{j}$ and h can now be obtained from Eqs. (16–24) and by the use of the following relation:

$$\langle G\delta[\boldsymbol{X}(t_1) - \boldsymbol{x}_1]W\rangle = G\langle\delta[\boldsymbol{X}(t_1) - \boldsymbol{x}_1]\rangle_c\langle W\rangle \tag{32}$$

where G can be $G_{jk}[\boldsymbol{x}_1, t_1; \boldsymbol{X}(t), t]$, $G_j[\boldsymbol{x}_1, t_1; T_d(t), t]$, or $G^\theta[\boldsymbol{x}_1, t_1; T_d(t), t]$ and where $\langle\delta[\boldsymbol{X}(t_1) - \boldsymbol{x}_1]\rangle_c$ is the conditional average[14] when $\boldsymbol{X}(t) = \boldsymbol{x}$, $\boldsymbol{V}(t) = \boldsymbol{v}$, and $T_d(t) = \theta$, that is,

$$\langle\delta[\boldsymbol{X}(t_1) - \boldsymbol{x}_1]\rangle_c = \langle\delta[\boldsymbol{X}(t_1) - \boldsymbol{x}_1] | \boldsymbol{X}(t) = \boldsymbol{x}, \boldsymbol{V}(t) = \boldsymbol{v}, T_d(t) = \theta\rangle \tag{33}$$

The final forms of the expressions are

$$\beta_v \langle u_i' W\rangle = -\left[\frac{\partial}{\partial x_k}\lambda_{ki} + \frac{\partial}{\partial v_k}\mu_{ki} + \frac{\partial}{\partial\theta}\omega_i - \gamma_i\right]\langle W\rangle \tag{34}$$

$$\beta_\theta \langle t' W\rangle = -\left[\frac{\partial}{\partial x_k}\Lambda_k + \frac{\partial}{\partial v_k}\Pi_k + \frac{\partial}{\partial\theta}\Omega - \Gamma\right]\langle W\rangle \tag{35}$$

where

$$\lambda_{ki} = \beta_v^2 \int_0^t \mathrm{d}t_1 \langle u_i'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{36}$$

$$\mu_{ki} = \beta_v^2 \int_0^t \mathrm{d}t_1 \langle u_i'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle \frac{\mathrm{d}}{\mathrm{d}t} G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{37}$$

$$\omega_i = \beta_v^2 \int_0^t \mathrm{d}t_1 \langle u_i'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G_j(\boldsymbol{x}_1, t_1; \theta, t) + \beta_v\beta_\theta \int_0^t \mathrm{d}t_1 \langle u_i'(\boldsymbol{x}, t) t'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G^\theta(\boldsymbol{x}_1, t_1; \theta, t) \tag{38}$$

$$\gamma_i = \beta_v^2 \int_0^t \mathrm{d}t_1 \left\langle \frac{\partial u_i'(\boldsymbol{x}, t)}{\partial x_k} u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\right\rangle G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{39}$$

$$\Lambda_k = \beta_v\beta_\theta \int_0^t \mathrm{d}t_1 \langle t'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{40}$$

$$\Pi_k = \beta_v\beta_\theta \int_0^t \mathrm{d}t_1 \langle t'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle \frac{\mathrm{d}}{\mathrm{d}t} G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{41}$$

$$\Omega = \beta_v\beta_\theta \int_0^t \mathrm{d}t_1 \langle t'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G_j(\boldsymbol{x}_1, t_1; \theta, t) + \beta_\theta^2 \int_0^t \mathrm{d}t_1 \langle t'(\boldsymbol{x}, t) t'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle G^\theta(\boldsymbol{x}_1, t_1; \theta, t) \tag{42}$$

$$\Gamma = \beta_v\beta_\theta \int_0^t \mathrm{d}t_1 \left\langle \frac{\partial t'(\boldsymbol{x}, t)}{\partial x_k} u_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\right\rangle G_{jk}(\boldsymbol{x}_1, t_1; \boldsymbol{x}, t) \tag{43}$$

Here the argument $(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)$ of variables u_i' and t' represents the value of these variables at time t_1 along a particle trajectory that passes through $(\boldsymbol{x}, \boldsymbol{v}, \theta)$ at time t. Various correlations of u_i' and t', having the form of $\langle b_i'(\boldsymbol{x}, t) b_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle$, appearing in Eqs. (36–43), represent the statistics of fluid variables fluctuations along the particle path, and their prediction in general flow situations remains a challenge.[28] It is possible to obtain governing partial differential equations for the correlations in the LHDI framework and that are difficult to solve.[13] Under certain situations, simplified expressions are obtained by Reeks[13] and Derevich[29] for these types of correlations by invoking the hypothesis of "independent averaging" suggested by Corrsin.[30] Later in this work, we will use one of the usual exponential forms[14,17] for the correlation having integral timescale $\tilde{T}_{b_i b_j} = 1/\beta_{b_i b_j}$, that is,

$$\langle b_i'(\boldsymbol{x}, t) b_j'(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_1)\rangle = \langle b_i'(\boldsymbol{x}, t) b_j'(\boldsymbol{x}, t)\rangle e^{\beta_{b_i b_j}(t_1 - t)} \tag{44}$$

The various integral timescales will be obtained by fitting exponential curves to the DNS data for different correlations. This exponential form for the correlation is valid for simple isotropic and homogeneous shear flows. The kinetic equation obtained here can be generalized to nonhomogeneous flows when the appropriate equations for the correlations applicable to such flows are used instead of Eq. (44). The closed kinetic equation is obtained by substituting Eqs. (34) and (35) in Eq. (8) and neglecting the terms in the square brackets on the RHS of Eq. (8). The closed expressions for $\boldsymbol{j}$ and h are exact when fluctuations $\boldsymbol{u}'$ and t' have Gaussian distributions.

The closed expressions for $\boldsymbol{j}$ and h generate certain exact analytical expressions in an ideal and simple situation of two-phase turbulent flow. These expressions can be considered as a first test for any other general closure scheme. That is, the general closure scheme is promising if it is capable of generating the exact expressions for the ideal situation. Now, we consider the ideal situation and obtain certain exact analytical expressions. In the ideal turbulent two-phase flow situation, $\bar{U}_i = 0$, $\bar{T} =$ constant, and the fluid velocity and temperature fluctuations are given by u_i' and t', respectively. We add a uniform translational velocity $\boldsymbol{u}_0$ and a uniform temperature θ_0 to each realization of turbulent fluid velocity and temperature, respectively. Furthermore, we assume that $\boldsymbol{u}_0$ and θ_0 have Gaussian distributions with zero mean and have finite correlation $\langle\boldsymbol{u}_0\theta_0\rangle$ over many realizations of the fluid velocity and temperature. In this situation, the closed expressions for $\boldsymbol{j}$ and h can be obtained by substituting $u_i' + u_{0i}$ and $t' + \theta_0$ in place of u_i' and t', respectively, in Eqs. (34–43). These equations can be simplified for the condition that u_i' and t' are not correlated to u_{0i} and θ_0 and by using

$$\langle u_{0i}(\boldsymbol{x}, t) u_{0j}(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_2)\rangle = (\delta_{ij}/3)\langle u_{0k}u_{0k}\rangle \equiv \delta_{ij}\langle u_0^2\rangle \tag{45}$$

$$\langle u_{0i}(\boldsymbol{x}, t)\theta_0(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_2)\rangle = \langle\theta_0(\boldsymbol{x}, t) u_{0i}(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_2)\rangle = \langle\theta_0 u_{0i}\rangle \tag{46}$$

$$\langle\theta_0(\boldsymbol{x}, t)\theta_0(\boldsymbol{x}, \boldsymbol{v}, \theta, t|t_2)\rangle = \langle\theta_0^2\rangle \tag{47}$$

along with the solutions of Eqs. (28–30) when $A_{jk} = C_\theta = 0$,

$$G_{jk} = \delta_{jk}(1/\beta_v)\left[1 - e^{-\beta_v(t - t_1)}\right] \tag{48}$$

$$G_j = 0 \tag{49}$$

$$G^\theta = e^{-\beta_\theta(t - t_1)} \tag{50}$$

These simplifications yield exact expressions for $\langle u_{0i}W\rangle$ and $\langle\theta_0 W\rangle$, written as

$$\langle u_{0i}W\rangle = -\langle u_0^2\rangle\left[\left(t + \frac{e^{-\beta_v t} - 1}{\beta_v}\right)\frac{\partial}{\partial x_i} + (1 - e^{-\beta_v t})\frac{\partial}{\partial v_i}\right]\langle W\rangle - \langle\theta_0 u_{0i}\rangle(1 - e^{-\beta_\theta t})\frac{\partial\langle W\rangle}{\partial\theta} \tag{51}$$

$$\langle\theta_0 W\rangle = -\langle\theta_0 u_{0i}\rangle\left[\left(t + \frac{e^{-\beta_v t} - 1}{\beta_v}\right)\frac{\partial}{\partial x_i} + (1 - e^{-\beta_v t})\frac{\partial}{\partial v_i}\right]\langle W\rangle - \langle\theta_0^2\rangle(1 - e^{-\beta_\theta t})\frac{\partial\langle W\rangle}{\partial\theta} \tag{52}$$

The expressions given by Eqs. (51) and (52) can be considered as a first test for the performance of other closure schemes in this ideal situation of nonisothermal two-phase flows.

Macroscopic Equations for Homogeneous Flows

Instead of directly solving the closed PDF equation by a numerical method, such as Monte Carlo, we adopt a different approach and obtain macroscopic equations governing the mean properties for the particle phase by taking various moments of the closed kinetic equation. Here, the macroscopic equations are written for the case of homogeneous flows where the spatial variation of statistical properties, other than the mean velocity and temperature, are zero.

The equations for the mean number density of the particle N, density weighted velocity $\bar{V}_j$, and temperature $\bar{\Theta}$, which are defined by

$$N = \int \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta \tag{53}$$

$$\bar{V}_j = \frac{1}{N} \int v_j \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta \tag{54}$$

$$\bar{\Theta} = \frac{1}{N} \int \theta \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta \tag{55}$$

are written as

$$\frac{\partial N}{\partial t} + N \frac{\partial \bar{V}_i}{\partial x_i} = 0 \tag{56}$$

$$\frac{\partial \bar{V}_j}{\partial t} + \bar{V}_i \frac{\partial \bar{V}_j}{\partial x_i} = \beta_v (\bar{U}_j - \bar{V}_j) \tag{57}$$

$$\frac{\partial \bar{\Theta}}{\partial t} + \bar{V}_i \frac{\partial \bar{\Theta}}{\partial x_i} = \beta_\theta (\bar{T} - \bar{\Theta}) \tag{58}$$

where the overbar denotes density weighted mean. The equations for higher-order correlations

$$\overline{v_i' v_j'} = \frac{1}{N} \int (v_i - \bar{V}_i)(v_j - \bar{V}_j) \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta$$

$$\overline{v_i' \theta'} = \frac{1}{N} \int (v_i - \bar{V}_i)(\theta - \bar{\Theta}) \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta$$

$$\overline{\theta' \theta'} = \frac{1}{N} \int (\theta - \bar{\Theta})(\theta - \bar{\Theta}) \langle W \rangle \, \mathrm{d}\boldsymbol{v} \, \mathrm{d}\theta$$

are written as

$$\frac{\partial \overline{v_j' v_n'}}{\partial t} = -\overline{v_i' v_j'} \frac{\partial \bar{V}_n}{\partial x_i} - \overline{v_i' v_n'} \frac{\partial \bar{V}_j}{\partial x_i} - 2\beta_v \overline{v_j' v_n'} - \lambda_{kj} \frac{\partial \bar{V}_n}{\partial x_k} - \lambda_{kn} \frac{\partial \bar{V}_j}{\partial x_k} + \mu_{jn} + \mu_{nj} \tag{59}$$

$$\frac{\partial \overline{v_j' \theta'}}{\partial t} = -\beta_v \overline{v_j' \theta'} - \beta_\theta \overline{v_j' \theta'} - \overline{v_i' v_j'} \frac{\partial \bar{\Theta}}{\partial x_i} - \overline{v_i' \theta'} \frac{\partial \bar{V}_j}{\partial x_i} - \lambda_{kj} \frac{\partial \bar{\Theta}}{\partial x_k} - \Lambda_k \frac{\partial \bar{V}_j}{\partial x_k} + \omega_j + \Pi_j \tag{60}$$

$$\frac{\partial \overline{\theta' \theta'}}{\partial t} = -2\beta_\theta \overline{\theta' \theta'} - 2\overline{v_i' \theta'} \frac{\partial \bar{\Theta}}{\partial x_i} - 2\Lambda_i \frac{\partial \bar{\Theta}}{\partial x_i} + 2\Omega \tag{61}$$

Equations (59–61) are computed and assessed against the DNS data for a case of homogeneous shear flow with constant $\partial \bar{U}_1 / \partial x_2$ and constant $\partial \bar{T} / \partial x_2$. In this case, the solutions of Eqs. (28–30) give various generalized response functions such as

$$G_{ij}(t_1; t) = G_{ij}^{(1)}(t_1; t) + (a_v / \beta_v) \delta(t_1 = 0) G_{ij}^{(1)}(t_1 = 0; t) \tag{62}$$

with

$$G_{ij}^{(1)}(t_1; t) = \frac{\delta_{ij}}{\beta_v} \left[1 - e^{-\beta_v (t - t_1)} \right] + \delta_{i2} \delta_{j1} \frac{\alpha}{\beta_v^2} \left\{ 2\left[e^{-\beta_v (t - t_1)} - 1 \right] + \beta_v (t - t_1) \left[1 + e^{-\beta_v (t - t_1)} \right] \right\} \tag{63}$$

$$G^\theta(t_1; t) = e^{-\beta_\theta (t - t_1)} + \frac{a_\theta}{\beta_\theta} \delta(t_1) e^{-\beta_\theta t} \tag{64}$$

$$G_j(t_1; t) = \delta_{j2} \int_{t_1}^{t} e^{-\beta_\theta (t - s)} \beta_\theta G_{22}(t_1; s) \frac{\partial \bar{T}}{\partial x_2} \, \mathrm{d}s \tag{65}$$

where $\alpha = \partial \bar{U}_1 / \partial x_2$.

For the case under consideration, $\gamma_i = \Gamma = 0$. Using the solutions for response functions and the exponential form given by Eq. (44) for various correlations of u_i' and t', Eqs. (36–38) and (40–42) give

$$\lambda_{ki} = \beta_v^2 \langle u_i'(\boldsymbol{x}, t) u_k'(\boldsymbol{x}, t) \rangle I_1 \left(\beta_{u_i u_k} \right) + \beta_v^2 \langle u_i'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle \delta_{k1} I_2 \left(\beta_{u_i u_2} \right) \tag{66}$$

$$\mu_{ki} = \beta_v^2 \langle u_i'(\boldsymbol{x}, t) u_k'(\boldsymbol{x}, t) \rangle J_1 \left(\beta_{u_i u_k} \right) + \beta_v^2 \langle u_i'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle \delta_{k1} J_2 \left(\beta_{u_i u_2} \right) \tag{67}$$

$$\omega_i = \beta_v^2 \langle u_i'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle K_1 \left(\beta_{u_i u_2} \right) + \beta_v \beta_\theta \langle u_i'(\boldsymbol{x}, t) t'(\boldsymbol{x}, t) \rangle K_2 \left(\beta_{u_i t} \right) \tag{68}$$

$$\Lambda_k = \beta_v \beta_\theta \langle t'(\boldsymbol{x}, t) u_k'(\boldsymbol{x}, t) \rangle I_1 \left(\beta_{t u_k} \right) + \beta_v \beta_\theta \langle t'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle \delta_{k1} I_2 \left(\beta_{t u_2} \right) \tag{69}$$

$$\Pi_k = \beta_v \beta_\theta \langle t'(\boldsymbol{x}, t) u_k'(\boldsymbol{x}, t) \rangle J_1 \left(\beta_{t u_k} \right) + \beta_v \beta_\theta \langle t'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle \delta_{k1} J_2 \left(\beta_{t u_2} \right) \tag{70}$$

$$\Omega = \beta_v \beta_\theta \langle t'(\boldsymbol{x}, t) u_2'(\boldsymbol{x}, t) \rangle K_1 \left(\beta_{t u_2} \right) + \beta_\theta^2 \langle t'(\boldsymbol{x}, t) t'(\boldsymbol{x}, t) \rangle K_2 (\beta_{tt}) \tag{71}$$

Here, various functions, namely, I_1, I_2, J_1, J_2, K_1, and K_2, are given by

$$I_1(\beta) = \int_0^t e^{-\beta(t - t_1)} G_{11}(t_1; t) \, \mathrm{d}t_1 = \frac{a_v}{\beta_v^2} e^{-\beta t} (1 - e^{-\beta_v t}) + \frac{e^{-(\beta_v + \beta) t} - 1}{\beta_v (\beta_v + \beta)} - \frac{e^{-\beta t} - 1}{\beta_v \beta} \tag{72}$$

$$\begin{aligned} I_2(\beta) &= \int_0^t e^{-\beta(t - t_1)} G_{21}(t_1; t) \, \mathrm{d}t_1 \\ &= \frac{a_v \alpha}{\beta_v^3} e^{-\beta t} \left[2(e^{-\beta_v t} - 1) + \beta_v t (1 + e^{-\beta_v t}) \right] \\ &\quad + \frac{\alpha}{\beta_v^2} \left\{ 2\left[\frac{1 - e^{-(\beta_v + \beta) t}}{\beta_v + \beta} + \frac{e^{-\beta t} - 1}{\beta} \right] + \frac{\beta_v}{\beta^2} [1 - e^{-\beta t}(1 + \beta t)] \right. \\ &\quad \left. + \frac{\beta_v}{(\beta_v + \beta)^2} \left[1 - e^{-(\beta_v + \beta) t} (1 + \beta_v t + \beta t) \right] \right\} \end{aligned} \tag{73}$$

$$J_1(\beta) = \int_0^t e^{-\beta(t - t_1)} \frac{\mathrm{d}}{\mathrm{d}t} G_{11}(t_1; t) \, \mathrm{d}t_1 = \frac{a_v}{\beta_v} e^{-(\beta_v + \beta) t} + \frac{1 - e^{-(\beta_v + \beta) t}}{\beta_v + \beta} \tag{74}$$

$$\begin{aligned} J_2(\beta) &= \int_0^t e^{-\beta(t - t_1)} \frac{\mathrm{d}}{\mathrm{d}t} G_{21}(t_1; t) \, \mathrm{d}t_1 \\ &= \frac{a_v \alpha}{\beta_v^2} e^{-\beta t} \left[1 - e^{-\beta_v t} (1 + \beta_v t) \right] \\ &\quad + \frac{\alpha}{\beta_v} \left\{ \frac{1 - e^{-\beta t}}{\beta} + \frac{e^{-(\beta_v + \beta) t} - 1}{\beta_v + \beta} \right. \\ &\quad \left. - \frac{\beta_v}{(\beta_v + \beta)^2} \left[1 - e^{-(\beta_v + \beta) t} (1 + \beta_v t + \beta t) \right] \right\} \end{aligned} \tag{75}$$

$$K_1(\beta) = \int_0^t e^{-\beta(t-t_1)} G_2(t_1; t)\,\mathrm{d}t_1$$

$$= \frac{1}{\beta_v(\beta_v - \beta_\theta)} \frac{\partial \bar{T}}{\partial x_2} \left[\frac{a_v}{\beta_v} e^{-\beta t}\left(\beta_v - \beta_\theta - \beta_v e^{-\beta_\theta t} + \beta_\theta e^{-\beta_v t}\right) + \frac{\beta_\theta - \beta_v}{\beta}(e^{-\beta t} - 1) + \frac{\beta_v}{\beta_\theta + \beta}\left(e^{-(\beta_\theta + \beta)t} - 1\right) - \frac{\beta_\theta}{\beta_v + \beta}\left(e^{-(\beta_v + \beta)t} - 1\right) \right] \tag{76}$$

$$K_2(\beta) = \int_0^t e^{-\beta(t-t_1)} G^\theta(t_1; t)\,\mathrm{d}t_1$$

$$= \frac{1}{\beta_\theta(\beta_\theta + \beta)} \left\{ [a_\theta(\beta + \beta_\theta) - \beta_\theta] e^{-(\beta + \beta_\theta)t} + \beta_\theta \right\} \tag{77}$$

Comparison with DNS Data

For the purpose of the assessment of the kinetic model, a DNS study is conducted at the limit of zero Mach number where the carrier phase is considered incompressible. In this manner, the energy equation can be decoupled from the continuity and momentum equations. Our numerical procedure for the carrier phase is similar to that adopted by Rogers et al.[31] who have considered the transport of a passive scalar. Here, we treat the carrier-phase temperature as a passive scalar that has no effect on the evolution of the velocity field, but not the other way around. The particles are assumed to be spherical and a modified Stokes drag is used to describe their transport by the carrier phase. Only low mass loading ratios are considered, so that the effects of the particles on the fluid can be neglected, that is, one-way coupling. We solve all of the equations in a transformed coordinate system using a Fourier spectral method with periodic boundary conditions in all of the directions. This transformation allows us to generate accurate statistics by averaging over the entire computational domain. (See Barré et al.[32] for more details.)

The DNS is performed for homogeneous shear flow with constant $\alpha = \partial \bar{U}_1/\partial x_2 = 2$ and constant $\partial \bar{T}/\partial x_2 = 2$ for particle time constants $\tau_d = 0.15$ and 0.3, specific heats ratio $\sigma = 1.0$, and Prandtl number $Pr = 0.7$. The fluid phase is simulated using 128^3 collocation points, and the statistics for particle related flow variables are calculated by averaging over the computed Lagrangian trajectories of 10^5 particles. The initial particle velocity and temperature are taken equal to the fluid velocity and temperature in the vicinity of particle at time $t = 0$. At this initial time, when the shear is applied, the fluid velocity field is isotropic. We have assured by checking the DNS data during the simulations that all of the scales of the flow have been properly resolved. The code implemented for this study is an extension of the code used in our previous study[7,33] and has undergone a variety of tests. The details of the DNS will be presented in a separate paper where various spectra and temporal evolution of different flow statistics are presented and carefully analyzed.

In the case of homogeneous shear flow with a constant mean temperature gradient, Eq. (56) gives $\partial N/\partial t = 0$ and Eqs. (57) and (58) give $\bar{V}_j = \bar{U}_j$ and $\bar{\Theta} = \bar{T}$, which is consistent with the DNS results. To assess the macroscopic equations (59–61), these are numerically integrated using a fourth-order accurate Runge–Kutta method. The integral timescale, $\tilde{T}_{u_i u_j}, \forall i, j$, is taken equal to $0.482k/\epsilon$ (Ref. 14). The values of $\tilde{T}_{u_i t}$, $\tilde{T}_{t u_i}$, and $\tilde{T}_{tt}$ are estimated from the DNS data for various correlations $\langle t'(\boldsymbol{x}, t)t'(\boldsymbol{x}, t)\rangle$, $\langle t'(\boldsymbol{x}, t)t'(\boldsymbol{x}, s)\rangle$, $\langle u_i'(\boldsymbol{x}, t)t'(\boldsymbol{x}, t)\rangle$, $\langle u_i'(\boldsymbol{x}, t)t'(\boldsymbol{x}, s)\rangle$, and $\langle t'(\boldsymbol{x}, t)u_i'(\boldsymbol{x}, s)\rangle$. The numerical solution requires temporal evolution of correlations $\langle u_i'(\boldsymbol{x}, t)u_j'(\boldsymbol{x}, t)\rangle$, $\langle u_i'(\boldsymbol{x}, t)t'(\boldsymbol{x}, t)\rangle$, and $\langle t'(\boldsymbol{x}, t)t'(\boldsymbol{x}, t)\rangle$ for the fluid phase and that are taken from the DNS data. Also, $a_v = a_\theta = 1$ because the initial velocity and temperature of the particles are the same as those of the fluid velocity and temperature in the vicinity of the particles.

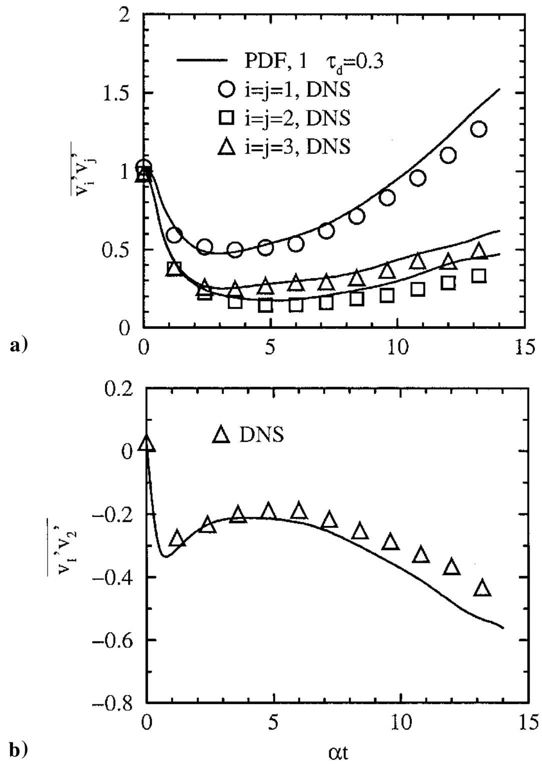

Fig. 1 Temporal evolution of particle Reynolds stresses.

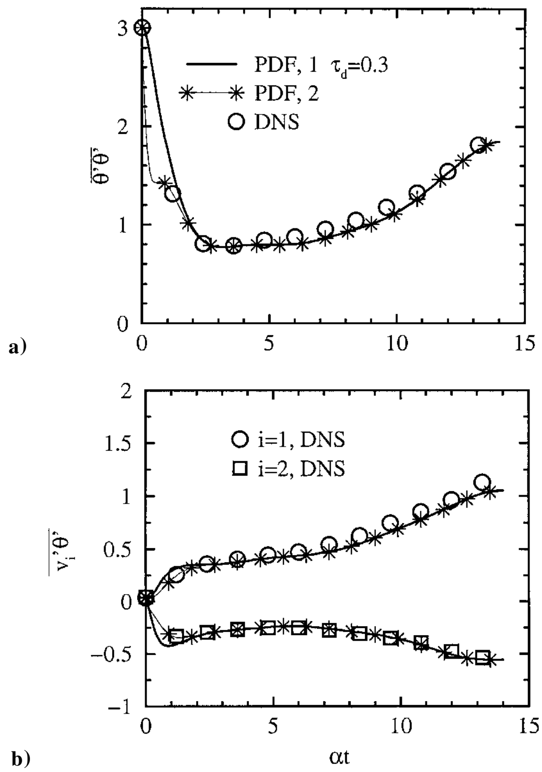

Fig. 2 Temporal evolution of particle temperature–temperature and velocity–temperature correlations.

The temporal evolutions of particle normal Reynolds stresses $\overline{v_1'v_1'}$, $\overline{v_2'v_2'}$, and $\overline{v_3'v_3'}$ are shown in Fig. 1a and of the shear component $\overline{v_1'v_2'}$ in Fig. 1b. The present model predictions are shown by solid lines and referred to as PDF 1. Figure 1 exhibits good agreement between model predictions and DNS data for $0 \leq \alpha t \leq 10$.

The temporal evolution of the temperature-related statistical properties of the particle phase are shown in Fig. 2. In Fig. 2, PDF 1 refers to the numerical solution of macroscopic equations when the initial condition effects, appearing through A_{jk} and C_θ, are taken into account. PDF 2 refers to the solution without the initial conditions. Figure 2a shows the comparison of PDF 1 and 2 solutions for $\overline{\theta'\theta'}$ with the DNS data. The inclusion of the initial conditions results in a better agreement with the DNS data, whereas without the initial conditions, $\overline{\theta'\theta'}$ is underpredicted initially. However, at later times, the PDF 1 and 2 solutions become identical. This suggests that initial conditions effects become insignificant after a certain period of time. Figure 2b shows the comparison of the predictions for $\overline{v_i'\theta'}$ with the DNS data and suggests a good agreement. Further comparisons (not shown) for $\tau_p = 0.15$ indicated that the overall performance of the models improve with the decrease of the particle time constant.

Conclusions

A kinetic equation for the nonisothermal particle or dispersed phase in two-phase turbulent flows has been derived by using the functional method. This equation is exact when the fluctuations of the fluid velocity and temperature along the particle path have Gaussian distributions. The macroscopic equations for various statistical properties of interest have been obtained for the case of homogeneous turbulent shear flow with constant mean velocity and temperature gradients. These equations have been numerically computed and the predictions have been compared against the DNS data for the purpose of assessment. The predictions have been found in good agreement with the data. This study suggests that the application of the functional method as a closure scheme is successful in obtaining the kinetic equation for nonisothermal flows. Nevertheless, the extent of its success remains to be established in cases of inhomogeneous flows where the fluctuations are not necessarily distributed as Gaussian.

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